

Our Friend the Logarithm

The logarithm of a number, N, is the power to which a base, 10, must be raised to give N. For example, $1000 = 10 \times 10 \times 10 = 10^3$, so the log of 1000 is 3. $100 = 10^2$ so its log is 2. Here is a short table of logs. Notice the logs of $N > 1$ are positive and logs for positive fractions are negative. The log of a negative number is undefined.

N	Log N
1000	3
100	2
10	1
1	0
0.1	-1
0.01	-2
0.001	-3

Finding the log of a number is finding the power to which 10 must be raised to give that number.

Taking the antilog means, given a log, what number corresponds to it. The anti-log of 3 is 1000 just as the log of 1000 is 3.

It is apparent that certain mathematical operations are possible with logs and antilogs. For example, to multiply two numbers, add their logs, then take the antilog.

That is, the log of $N \times M = \log N + \log M$.

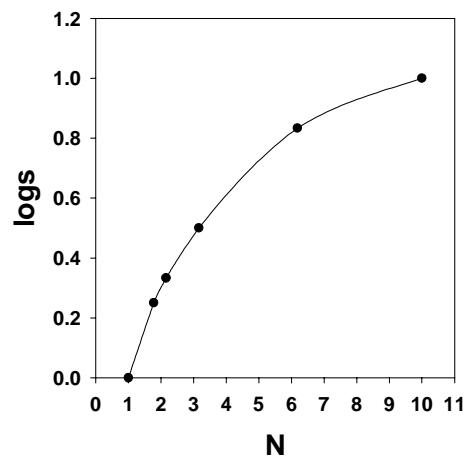
For example, to multiply 100×0.1 you could say $100 \times 0.1 = X$, and the log of $X = \log N + \log M$. This would give $\log X = (2 + -1) = 1$. If the $\log X = 1$ $X =$ antilog of 1 which is 10. We can see by inspection that this is the correct answer because 100×0.1 is certainly 10. Needless to say, things can get more complicated.

Another simple observation is that $\log N = -\log (1/N)$. For example if $N = 100$ it is clear that its log is 2. The log of $(1/N = 1/100 = 0.01)$ is -2 and we can see that the relation is true, since $2 = -(-2)$.

Now the table above, and our examples have all been simple, since we restricted ourselves to number that were simple powers of 10. Most numbers are not like that. From the table it is clear that $\log 1 = 0$ and $\log 10 = 1$, so numbers like 3 must have logs that are greater than 0 but less than 1. What are they? Well, some smart guys have figured that all out and put the logs in more detailed tables and into your calculators, so in some sense you only need to work with them. It is easy to get an idea of how the logs for numbers between 1 and 10 were discovered though.

A little reflection will reveal that the log of the square root of 10, that is 3.162... must be 0.5. The reason is clear. We know from above that if we multiplied 3.162×3.162 it must give 10 and we know the $\log 10$ must = $\log 3.162 + \log 3.162$. Since the $\log 10$ is 1, the other two logs must each be 0.5 so they can sum to 1.0. In the same way, the cube root of 10, that is 2.154... must have a $\log = 0.3333...$ and the fourth root, 1.778, must have a $\log = 0.25$. Now we could fool with these to get more. For example, multiply 3.162×2.154 to get 6.82, and we know IT'S log must be $0.5 + 0.333 = 0.833$. You can fiddle around like this to get as many logs as you want, but if you simply PLOT

the data we have so far, you get a decent log curve, shown below. You could, in principle, read off the logs from this curve.



Usually, logs for numbers between 1 and 10 are tabulated, as shown below. The integers are down the left side, fractions across the top, and each cell contains the log, with an implied decimal point. For example, $\log 2 = 0.301$, the $\log 3 = 0.477$, and the $\log 6.4$ is 0.806 . This information is also stored, or generated, in your calculator.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	000	041	079	113	146	176	204	230	255	278
2	301	322	342	361	380	397	414	431	447	462
3	477	491	505	518	531	544	556	568	579	591
4	602	612	623	633	643	653	662	672	681	690
5	698	707	716	724	732	740	748	755	763	770
6	778	785	792	799	806	812	819	826	832	838
7	845	851	857	863	869	875	880	886	892	897
8	903	908	913	919	924	929	934	939	944	949
9	954	959	963	968	973	977	982	986	991	995

You can use this to find the log of any number. For example, what is the log of 330? The table above shows that N , 300, lies between 100 1000 so its log must be greater than 2 but less than 3, but what is it? Well, $300 = 3 \times 100$ so the \log of $300 = \log 3 + \log 100 = 0.477 + 2.0 = 2.477$. That is $300 = 10^{2.477}$.

To take the antilog of 2.477 break it into an integer and a fraction (mantissa) which is the log of a number between 1 and 10. That is $2.477 = 2 + 0.477$, the separate antilogs are 100 and 3 respectively so the antilog must be $100 \times 3 = 300$. Your calculator is so smart it will just take the antilog of 2.477, so you don't have to worry about it.

In chemistry we often take antilogs of negative logs. For example, we might know $\text{pH} = -\log [\text{H}^+] = 2.7$. But what is $[\text{H}^+]$? Well, $-\log [\text{H}^+] = 2.7$ so $\log [\text{H}^+] = -2.7$. From the first short table that means $[\text{H}^+]$ lies between 0.001 and 0.01. To be more specific take antilog of -2.7 . This is no trouble for your calculator. By hand, we need to convert -2.7 to an integer and a positive fraction, and that will be $-3 + 0.3$. The antilog of 0.3 is 2 (from table above) and antilog of -3 is 10^{-3} . Therefore the antilog of -2.7 is $2 \times 10^{-3} = [\text{H}^+]$.